Conceptual Summary for Mechanics

This is meant to be a quick and dirty summary of the conceptual points presented during the first semester in AP Physics (that is, while we were studying Classical Newtonian Physics). It is probably not definitive, but it hits most of the high points. Few equations are being used because this year's test is going to be a prose special, and formulas are not going to be as important as understanding the underlying physics.

Kinematics:

--if the acceleration in a particular direction is constant, there are relationships that are always true—the kinematic relationship;

--acceleration is the rate at which velocity changes with time;

--velocity is the rate at which position changes with time;

--average velocity is the one velocity that, if traveled at over the time interval in question, will take you through the prescribed displacement over that period of time;

--average acceleration is the one acceleration that, if applied over the time interval in question, will change the body's velocity in the prescribed manner;

--you can have a constant velocity in one dimension and not in another—2-d projectile motion is a good example with v_x constant because there is typically assumed to be no acceleration in the x-direction (unless the body is wearing a jet pack or there is drag) and v_y not constant as the acceleration of gravity (and possibly drag) acts in the y-direction;

Newton's Laws:

--the net force acting on an object in a particular direction is proportional to the acceleration in that direction, with the proportionality constant being the body's mass;

--mass is a relative measure of a body's inertia, its *resistance to changing its motion*;

--force is a vector;

--a free body diagram identifies all the forces acting on a body;

--one axis on a f.b.d. should be defined along the line of the body's acceleration;

--there are four naturally occurring forces in the Newton's Second Law pantheon, tension, gravity, normal and friction (and a fifth, the push-me, pull-you if you want to include the possibility of a non-descript force that is there but you don't know its origin);

--tension forces (T or F_T) provided by strings or ropes always act away from the body feeling their effect;

--gravitational forces (mg or F_{mg}) provided by the earth or other celestial body always act downward (toward the center of the body);

--normal forces (N or F_N) are forces of support and always act perpendicularly outward away from the body providing the force;

--frictional forces come in two types, static and kinetic;

--static friction (f_s or F_{f_x}) is produced when two bodies are up against one another and one of the bodies tries to move relative to the other—it is a consequence of the atomic interaction between the two body's surfaces; it is a kind of drag effect that holds the bodies stationary to one another;

--the *maximum* static frictional force is proportional to the normal force between the two bodies, with the proportionality constant being the coefficient of static friction (μ_s);

--it is possible to have a system in which there is static friction, but the static friction is not maximum;

--kinetic friction (f_k or F_{f_k}) is produced when two bodies are up against one another and are moving (sliding) relative to the other—it is a consequence of the atomic interaction between the two body's surfaces;

--with kinetic friction, if one of the bodies is station (a tabletop or a wall) and the other is sliding relative to it, kinetic friction will act as a drag, will be opposite the direction of the relative motion, and will tend to slow the body down;

--in all cases, the direction of kinetic friction is opposite the direction of the *relative motion* between the two bodies

--the *maximum* static frictional force is proportional to the normal force between the two bodies, with the proportionality constant being the coefficient of static friction (μ_s);

--the direction of static frictional force on a body is opposite the direction the body would accelerate *if it broke loose* and moved;

--a spring can produce a force on a body shoved up against it (or attached to it) if the spring and body are displaced from the system's equilibrium position (normally characterized by *x*);

--the force a spring provides to an attached body will be proportional to the displacement of the spring, or $\vec{F}_{spring} = (kx)(-\hat{i}) = -kx\ \hat{i}$, where *k* is the spring's spring constant;

--a spring's spring constant *k* is defined as the amount of force required to elongate or compress the spring per meter; it is always positive and its value tells you how stiff the spring is;

--forces or their components that act along (or opposite) the line of motion motivate a body to accelerate so as to pick up speed or slow down (they change the *magnitude* of the body's velocity vector);

--forces that act perpendicular to the line of motion motivate a body to accelerate so as to change the *direction* of the velocity vector;

--forces that change the direction of the velocity vector are called *centripetal forces*;

--a centripetal force is not a new kind of force, it is not like tension or gravity or normal or friction;

--the word *centripetal* is used to identify one or a combination of the four normally occurring forces in a system (tension, gravity, etc.) or their components, that is/are doing a special thing—that is/are pushing the body out of straight-line motion;

--forces that act centripetally, being perpendicular to the motion do no work (that's why they don't make bodies pick up speed or slow down . . .);

--on a f.b.d., the centripetal direction will always be along a line between the body and the *center of the arc upon which the body is moving*;

Energy considerations:

--the work done on a body by a force as the body moves some distance is mathematically defined as the dot product of the force and displacement vector, or $W_F = \vec{F} \bullet \vec{d}$ (that is, it's the component of force along the line of displacement times the displacement, with a positive or negative sign thrown in to identify if the work is increasing the body's speed or slowing it down);

--if a force field is not constant, or if the force does not follow a straight-line path, determine the work the field does as a body moves from one point to another requires a modification of that dot product shown above; that modification is: $W = \int_{x_1}^{x_2} \vec{F} \bullet d\vec{r}$, where $d\vec{r}$ is a differential bit of the path traversed;

--positive work is associated with putting energy into a system, motivating a body to increase its speed; negative work is associated with taking energy out of a system, motivating a body to decrease its velocity;

--when a non-zero amount of work is done on a body, the body's kinetic energy (which is to say, it's velocity) changes (this is the work/energy theorem, or $W_{net} = \Delta KE$);

--if the amount of work a force field does on a body as the body moves through the field is *path independent,* the field is said to be *conservative*;

--the work done by friction is non-conservative;

--the work done by gravity near the surface of the earth, and by an ideal spring, is conservative;

--because work done in the a conservative force field isn't predicated on the path the body takes in getting from its start to end point, a function associated with the force field can be derived that is end-point dependent and that allows the calculation of the work done by the field as the body moved between those two points; such a function is called a *potential energy function* and it's defined by $W = -\Delta U$;

--put a little differently, a potential energy function is a contrived mathematical function with one use and one use only, to allow the user to determine the amount of work the function's associated force field does as a body moves from one point in the field to another;

--as potential energy functions are defined $W = -\Delta U$ and work quantities are defined as $W = \int_{x_1}^{x_2} \vec{F} \bullet d\vec{r}$, the relationship between a known force and its potential energy difference is seen to be $\Delta U = -\int_{x_1}^{x_2} \vec{F} \bullet d\vec{r}$;

-- $\Delta U = -\int_{x_1}^{x_2} \vec{F} \bullet d\vec{r}$ is the relationship used to derive the potential energy function that goes with a particular force function, where the integral's limits go from where the force is zero (if such a point exists) to an arbitrary point in the field;

--the derived gravitational potential energy function when close to the earth is *mg*;

--the derived spring potential energy function is $\frac{1}{2}kx^2$, where *k* is the spring constant and *x* is the spring's displacement from its equilibrium position;

--taking $\Delta U = -\int_{x_1}^{x_2} \vec{F} \bullet d\vec{r}$ and manipulating, it can be seen that the relationship between a known potential energy function and it's unknown force function is $\vec{F} = -\vec{\nabla} U$, where the del operator $\vec{\nabla}$ is just the spatial derivatives of U taken in the various directions, times their associated unit vectors (you'll only have to worry about this in one dimension);

--in one-dimension using cartesian coordinates, the above relationship becomes $\vec{F} = -\frac{\partial U}{\partial x}\hat{i}$, or $F_x = -\frac{\partial U}{\partial x}$;

--in one-dimension using polar spherical coordinates, the above relationship becomes $\vec{F} = -\frac{\partial U}{\partial r}\hat{r}$, or $F_r = -\frac{\partial U}{\partial r}$;

--the bottom line is that the magnitude of a force field F at a particular point is equal to the spatial rate-of-change of the field's potential energy function $\frac{\partial U}{\partial x}$ evaluated at the point, with a minus sign placed out in front;

--the work/energy theorem can be manipulated into the modified conservation of energy relationship $\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$, which is nice because each bailiwick *tells* you what to look for . . . (i.e., $\sum KE_1$ says "is the body moving at the beginning of the interval . . . if no, write 0, is yes, write $\frac{1}{2}mv_1^{\ 2}$," etc.);

Momentum:

--a force $\vec{F}$ applied over a period of time Δt will exert an impulse $\vec{F}\Delta t$ on the object that will change the object's momentum such that $\vec{F}\Delta t = \Delta\vec{p}$, where the momentum vector $\vec{p} = m\vec{v}$;

--put a little differently, a body's *momentum* gives you a feel for whether a relatively large or small force would be required to bring the body to rest in a given (smallish) amount of time;

--an internal impulse is an impulse that is generated by the interaction of the pieces of the system (example: impulses generated by the forces produced when two balls hit one another);

--in a particular direction, if all the forces acting on a system generate internal impulses, then the net momentum of that system in that direction will be conserved (will not change with time)—that means the sum of the momenta (signs included) in that direction at the beginning of an interval will equal the sum of the momenta in that direction at the end of the interval;

--put a little differently, if there are no externally impulses in a particular direction acting over a time interval (or if the external forces are small and the time interval is, likewise, small), momentum will be conserved in the direction *through the interval*;

--momentum can be conserved in one direction and not in another (projectile motion is a good example—no external impulse in the x-direction while gravity acts as an external impulse in the y-direction);

-- in one-dimensional collisions where objects can respond freely, even if a small external impulse is present over the time interval of the interaction, the net momentum *just before the collision* will equal the net momentum *just after the collision*—this is referred to as momentum being conserved *through the collision*;

--an inelastic collision is an "normal" collision in which *energy is not conserved*;

--a perfectly inelastic collision is an elastic collision (energy not conserved) in which the two objects become one after the collision);

--an elastic collision is one in which *energy is assumed to be conserved*;

--note: because collisions usually occur quickly, potential energy doesn't generally change appreciably over the collision's tiny time interval so when energy is conserved in an elastic collision, it is *kinetic energy* that is conserved through the collision);

--it is not uncommon to have problems in which you have to decide when you can legitimately use energy considerations and when you can legitimately use momentum considerations;

Rotational Motion:

--for every translational parameter, there is a rotational parameter: that means:
$x \Rightarrow \theta$ (angular position); $v \Rightarrow \omega$ (angular velocity); $a \Rightarrow \alpha$ (angular acceleration);
$m \Rightarrow I$ (moment of inertia); $\vec{F} \Rightarrow \vec{\tau}$ (torque); $\vec{p} \Rightarrow \vec{L}$ (angular momentum);

--a point r units away from a fixed point, moving with angular velocity ω about that fixed point, will have an translational velocity that is proportional to the distance out, or $v = r\omega$;

--a point r units away from a fixed point, moving with angular acceleration α about that fixed point, will have an translational acceleration that is proportional to the distance out, or $a = r\alpha$;

--the *translational velocity* of every point on a rolling object is different, but the *angular velocity* about each of those points will be the same (that is the beauty of rotational parameters in a rotating setting);

--the instantaneous *translational velocity* of the contact point on a rolling object is zero;

--the four points listed above are the justification for the observation that for a rolling object of radius R, the velocity *v* of the body's center of mass will be related to the angular velocity of the body about its center of mass by $v_{cm} = R\omega$;

--and by the same logic, the acceleration of the body's center of mass is related to the body's angular acceleration as $a_{cm} = R\alpha$;

--*rotational kinematics* only is applicable when the angular acceleration of a system is constant (just as was the case with *translational kinematics*–constant acceleration was required);

--*rotational kinematics* work just like translational kinematics—draw a picture; put in the parameters you know; identify what you're looking for; find a *rotational kinematic equation* that has what you know and what you're looking for;

--an angular velocity vector (or any rotational vector) has a direction defined *perpendicular to the plane* in which the action occurs (example: if the motion is in the x-y plane, the "direction" of the angular velocity or torque vectors will be in the z-direction, or using unit vector notation, in the $\hat{k}$ direction, positive or negative depending . . . see next entry);

--the kind of rotational motion problems you will deal with in this course are technically one-dimensional in that the action happens in one plane only (usually the x-y plane)—so although you will have to include the + and – signs with angular velocities and torque quantities (both vector quantities), you will not need to include the $\hat{k}$ unit vector when dealing with those vectors as *everything* will have a $\hat{k}$ unit vector attached to it;

--positive or negativeness of an angular velocity vector is defined by the right-hand rule: if your right-hand circles in the direction of motion, and if it is in the counterclockwise direction (or instance), your thumb will point *out of the page* in the $+\hat{k}$ direction. This angular velocity vector would be termed *positive*. If the right-hand's rotation placed the thumb pointing *into* the page in the $-\hat{k}$ direction, the vector would be denoted as negative;

--the rotational counterpart of force is torque;

--the torque about a point due to a force is related to the distance out from the point (defined by a position vector) and the perpendicular component of the force vector producing the impetus;

--put differently, the torque about a point due to a force is related to the size of the force producing the rotational impetus, the distance out from the point (defined by a position vector) and the sine of the angle between the force vector and the position vector . . . in other words, the cross product of $\vec{r}$ and $\vec{F}$;

--for equilibrium situations (rigid body problems), the sum of the forces in any direction must equal zero and the sum of the torques about any point must equal zero;

--for situations in which there is rotation but no translation, the net torque acting on a body will be proportional to the angular acceleration of the body, with the proportionality constant being the rotational inertia of the body (the body's moment of inertia I)—this is essentially Newton's Second Law, rotation style;

--a body's rotational inertia, or *moment of inertia*, is related to the body's mass and how its mass is distributed about the axis of interest;

--the *moment of inertia* about a fixed point for a point mass r units out is mr^2;

--the *moment of inertia* about an axis through a body's center of mass will be a minimum; it will increase as the axis moves away from the center of mass;

--if you know the moment of inertia about an axis through a body's center of mass, and you want the *moment of inertia* about an axis parallel to that known axis and a distance *d* units from it, the *parallel axis theorem* will allow you to determine that new moment of inertia; it will be $I_p = I_{cm} + md^2$;

--for situation in which there is both rotation and translation, the translational version of N.S.L. is applicable and the rotational version of N.S.L is applicable independent of one another;

--the link between the rotational and translational N.S.L. relationships is usually $a_{cm} = R\alpha$;

--translating and rotating problems can either be approached from the perspective of what's happening to the body's center of mass (the body's center of mass is translationally accelerating while its mass is angularly accelerating about the center of mass) or from the perspective of a pure rotation about the contact point (which is instantaneously stationary);

--the basics of energy considerations in rotating systems is the same as before; it is still true that over any interval, $\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$;

--rotating objects have rotational kinetic energy equal to $\frac{1}{2}I\omega^2$;

--there are no rotating potential energy functions;

--you can determine the work a torque does as a body rotates through an angular displacement using the rotational counterpart to $\vec{F} \bullet \vec{d}$; that is, $\pm\tau\left(\Delta\theta\right)$, where the sign depends upon whether the torque is in the same direction as the angular displacement or opposite the direction of the displacement;

--unlike N.S.L. situations, you can have rotational and translational energy expressions all in one equation (put a little differently, $\frac{1}{2}mv^2$ and $\frac{1}{2}I\omega^2$ have the same units);

--a torque $\vec{\tau}$ applied over a period of time Δt will produce a rotational impulse $\vec{\tau}\Delta t$ on the object that will change the object's *angular momentum L* such that $\vec{\tau}\Delta t = \Delta\vec{L}$, where the *angular momentum vector* $\vec{L} = I\vec{\omega}$;

--(note that if the body is a point mass moving with translational momentum $\vec{p}$, the angular momentum vector can be determined using $\vec{L} = \vec{r}x\vec{p}$);

--put a little differently, a body's *angular momentum* gives you a feel for whether a relatively large or small torque would be required to bring the rotating object to rest in a given (smallish) amount of time;

--just as the net force acting on an body is equal to the *time rate of change* of the body's momentum ($\vec{F} = \frac{d\vec{p}}{dt}$), the net torque acting on a body is equal to the *time rate of change* of the body's *angular momentum* ($\vec{\tau} = \frac{d\vec{L}}{dt}$);

--if there are no externally, torque-related impulses acting over a time interval (or if the external torques are small and the time interval is, likewise, small), angular momentum will be conserved *through the interval*;